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OUTLIERS IN MULTIVARIATE GARCH MODELS

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Abstract

Outliers of moderate magnitude cause large changes in financial time series of prices and returns and affect both the estimation of parameters and volatilities when fitting a GARCH-type model. The multivariate setting is still to be studied, but similar biases and impacts on correlation dynamics are believed to exist. The accurate estimation of the correlation structure is crucial in many applications, such as portfolio allocation and risk management. This paper focuses on these issues by studding the impact of additive outliers (isolated, patches and volatility outliers) on the estimation of correlations when fitting well known multivariate GARCH models and by proposing a general detection algorithm based on wavelets that can be applied to a large class of multivariate volatility models. This procedure can be also interpreted as a model miss-specification test since it is based on residual diagnostics. The effectiveness of the new proposal is evaluated by an intensive Monte Carlo study before it is applied to daily stock market indices. The simulation studies show that correlations are highly affected by the presence of outliers and that the new method is both effective and reliable, since it detects very few false outliers.

Keywords: Additive Outliers; Correlations; Volatilities; Wavelets.

JEL classification: C10; C13; C53; C58; G17.

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Outliers in Multivariate GARCH Models: Effects and Detection*

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June 16, 2014

Abstract

Outliers of moderate magnitude cause large changes in financial time series of prices and returns and affect both the estimation of parameters and volatilities when fitting a GARCH-type model. The multivariate setting is still to be studied, but similar biases and impacts on correlation dynamics are believed to exist. The accurate estimation of the correlation structure is crucial in many applications, such as portfolio allocation and risk management. This paper focuses on these issues by studding the impact of additive outliers (isolated, patches and volatility outliers) on the estimation of correlations when fitting well known multivariate GARCH models and by proposing a general detection algorithm based on wavelets that can be applied to a large class of multivariate volatility models. This procedure can be also interpreted as a model miss-specification test since it is based on residual diagnostics. The effectiveness of the new proposal is evaluated by an intensive Monte Carlo study before it is applied to daily stock market indices. The simulation studies show that correlations are highly affected by the presence of outliers and that the new method is both effective and reliable, since it detects very few false outliers.

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1 Introduction

The correlation structure of security returns is the keystone of both portfolio allocation and risk management decisions. In the literature, there are several models to estimate correlations, being the multivariate GARCH the most popular class of models. Financial series of returns often exhibit excess of kurtosis that can be caused by large unexpected observations. In the univariate context, some authors tried to capture this excess of kurtosis by estimating

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volatility models with fat tail distributed errors. However, it was observed that the estimated residuals of these models still registered excess kurtosis (see Baillie and Bollerslev, 1989; Teräsvirta, 1996). An alternative approach considers that some observations are outliers that can affect the estimation of parameters (Fox, 1972; Van Dijk et al., 1999; Verhoeven and McAleer, 2000), the tests of conditional homoscedasticity (Carnero et al., 2007; Grossi and Laurini, 2009), the out-of -sample volatility forecasts (Ledolter, 1989; Chen and Liu, 1993; Franses and Ghijsels, 1999; Grané and Veiga, 2010; Boudt et al., 2013), the volatility estimates (Carnero et al., 2012) and the risk measures (Grané and Veiga, 2014). All these works have studied these effects on univariate GARCH-type models. It is well known that, when there is a positive additive outlier in the level of the return series, standard GARCH models tend to overestimate the volatility the days following the presence of the outlier. Similar biases are believed to occur when the volatilities and correlations are estimated using multivariate volatility models of the GARCH family. However, to the best of our knowledge, such believe is still to be studied.

The first objective of this paper is to study the effect of additive outliers (isolated level outliers, patches of level outliers and volatility outliers) on the estimated correlations of multivariate GARCH models.[1] We focus on the diagonal Baba-Engle-Kraft-Kroner (D-BEKK) by Engle and Kroner (1995), the conditional constant correlation (CCC) model by Bollerslev (1990) and the dynamic conditional correlation (DCC) model by Engle (2002). We have chosen these models because they are often used in empirical works (see Bauwens et al., 2006; Silvennoinen and Teräsvirta, 2009, for excelent surveys on these models). Yet, the conclusions and the procedures of this work can be extended to other more sophisticated multivariate volatility models. The second aim is to propose an outlier detection procedure for multivariate volatility models based on wavelets that can also be interpreted as a misspecification test to the model. The procedure is based on the multivariate series of residuals and if outliers are detected in these series this may suggest the rejection of the model.

The Monte Carlo study leads us to conclude that outliers affect the estimated correlations and this effect is stronger for the conditional correlation models (CCC and DCC). Second, our detection procedure is very reliable, not only because the percentage of correct detections is quite high, specially for additive level outliers, but also because it detects very few false outliers. This property ensures that when one observation is detected as a possible outlier, it is indeed an outlier.

The advantages of our method are several: first, it can be applied to any multivariate volatility model given that the errors follow a known distribution, second it is well suited for detecting isolated single/multiple outliers and patches of outliers; third, the method is easy and quick to apply, which makes it in an attractive tool for use by academic communities and/or by practitioners; fourth, it can be a applied to a high number of series, and finally, it is reliable since it detects very few false outliers.

The organization of this paper is as follows. In Section 2 we present the volatility models used in the paper and review two types of additive outliers introduced by Hotta and Tsay (1998). In Section 3 we study the effect of outliers on the estimated correlations via a intensive simulation study. In Section 4 we present and evaluate the performance of the algorithm for outlier detection and we apply it to three daily stock market indices in Section 5. Finally, we conclude in Section 6.

[1] See Galeano and Peña (2013) for a resume on the different types of outliers.

2 Additive outliers in multivariate volatility models

Multivariate financial time series of returns exhibit similar patterns to those of univariate series, such as, persistent time-varying volatilities. Additionally, they display time-varying correlations that are often modeled by multivariate GARCH models. One advantage of these models is that they are flexible enough to represent the dynamics of the volatilities and correlations.

In this section we empirically evaluate the effect of outliers on the estimation of correlations. We consider three models: the diagonal Baba-Engle-Kraft-Kroner (D-BEKK) model defined in Engle and Kroner (1995), constant conditional correlation (CCC) model by Bollerslev (1990), and the dynamic conditional correlation (DCC) model by Engle (2002). These three models are pioneer in the financial econometrics literature and are often applied empirically to many fields such as portfolio management, asset allocation, volatility spillover transmission, contagion, etc. However, the methodology developed in this paper (to be described in Section 4) is not restricted to these models.

Let $\{\mathbf{y}_t\}$ be a vector stochastic process with dimension $N \times 1$ such that $E(\mathbf{y}_t) = \mathbf{0}$ and $\mathcal{F}_{t-1}$ is the information set till time $t-1$. We consider that $\mathbf{y}_t = \boldsymbol{\varepsilon}_t$ and $\boldsymbol{\varepsilon}_t = \mathbf{H}_t^{1/2}\boldsymbol{\eta}_t$, where $\mathbf{H}_t$ is the conditional covariance matrix of $\mathbf{y}_t$ and $\boldsymbol{\eta}_t$ is an iid vector error process such that $E(\boldsymbol{\eta}_t\boldsymbol{\eta}_t') = \mathbf{I}$, the identity matrix of order N. We assume that there is no linear dependence in $\mathbf{y}_t$. Different approaches in the literature propose different models for the dependence of $\mathbf{H}_t$ on past information $\mathcal{F}_{t-1}$.

In the D-BEKK, this dependence of $\mathbf{H}_t$ on past information is modeled directly. In contrast, in the CCC and DCC models, which belong to a subclass of the multivariate GARCH models called conditional correlation models, first the conditional variances are modeled using univariate specifications and then $\mathbf{H}_t$ is obtained by using these conditional standard deviations together with some specifications of the correlations (constant for CCC and time-varying for DCC).

We now proceed to a more detailed description of these three models.

Diagonal BEKK model The D-BEKK is a restricted version of the model defined in Engle and Kroner (1995), where $\mathbf{H}_t$ dependence on past information is modeled as follows:

$$\mathbf{H}_t = \mathbf{C}\mathbf{C}' + \mathbf{A}'\boldsymbol{\varepsilon}_{t-1}\boldsymbol{\varepsilon}_{t-1}{}'\mathbf{A} + \mathbf{B}'\mathbf{H}_{t-1}\mathbf{B}, \tag{1}$$

where $\mathbf{A}$ and $\mathbf{B}$ are $N \times N$ diagonal matrices and $\mathbf{C}$ is a $N \times N$ lower triangular matrix. The D-BEKK is covariance stationary if and only if $a_{ii}^2 + b_{ii}^2 < 1$ for all i, where a_{ii} and b_{ii} are, respectively, the diagonal elements of $\mathbf{A}$ and $\mathbf{B}$.

Conditional correlation models The CCC model is given by

$$\mathbf{H}_t = \mathbf{D}_t\mathbf{R}\mathbf{D}_t = \left(\rho_{ij}\sqrt{h_{ii,t}\,h_{jj,t}}\right)_{ij,t},$$

where $\mathbf{D}_t = \mathrm{diag}(h_{11,t}^{1/2}, ...h_{NN,t}^{1/2})$. Here $h_{ii,t}$ is defined as a univariate GARCH–type model such that $h_{ii,t} = \alpha_{0i} + \alpha_{1i}\varepsilon_{i,t-1}^2 + \beta_{1i}h_{ii,t-1}$ and $\mathbf{R} = (\rho_{ij})_{1\le i,j\le N}$ is a symmetric positive definite matrix, with $\rho_{ii} = 1$ and $\rho_{ij} = \rho_{ji}$ for $i,j = 1,\dots,N$. If the N conditional variances are positive and $\mathbf{R}$ is a positive define matrix then $\mathbf{H}_t$ is positive definite. The number of parameters to be estimated are $N(N+5)/2$. Furthermore, in the univariate GARCH models

$\alpha_{0i} > 0$, $\alpha_{1i} \geq 0$ and $\beta_{1i} \geq 0$ to guarantee positive conditional variances and $\alpha_{1i} + \beta_{1i} < 1$ to enforce stationary (see Duan et al., 2006).
On the other hand, the dynamic conditional correlation model, DCC, by Engle (2002) is defined as

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t \tag{2}$$

with $\mathbf{D}_t$ defined as before and $\mathbf{R}_t = \left(q_{ij,t} / \sqrt{q_{ii,t}\, q_{jj,t}} \right)_{ij,t}$, where $\mathbf{Q}_t = (q_{ij,t})$ is a $N \times N$ symmetric positive definite matrix given by:

$$\mathbf{Q}_t = (1 - \alpha - \beta)\, \bar{\mathbf{Q}} + \alpha\, \mathbf{u}_{t-1}\mathbf{u}'_{t-1} + \beta\, \mathbf{Q}_{t-1}, \tag{3}$$

where $\mathbf{u}_t = (u_{1,t}, \ldots, u_{N,t})'$ with $u_{i,t} = \varepsilon_{i,t} / \sqrt{h_{ii,t}}$, $\bar{\mathbf{Q}}$ is the unconditional variance matrix of $\mathbf{u}_t$ and α and β are non-negative scalar parameters that satisfy $\alpha + \beta < 1$ (see Bauwens et al., 2006).
We now proceed to define the type of outliers we are going to study. Following Doornik and Ooms (2005) and Grané and Veiga (2010), for the univariate case, we distinguish two type of additive outliers, level and volatility, and propose a simple extension to the multivariate case.

2.1 Additive level outliers

Additive level outliers (ALOs) can be caused by institutional changes or market corrections that do not affect volatility. In this case, the conditional mean equation of the multivariate volatility model becomes:

$$\begin{aligned} \mathbf{y}_t &= \boldsymbol{\omega} \cdot I_T(t) + \boldsymbol{\varepsilon}_t \\ \boldsymbol{\varepsilon}_t &= \mathbf{H}_t^{1/2} \boldsymbol{\eta}_t, \end{aligned} \tag{4}$$

where $\boldsymbol{\eta}_t$ is as before, that is, an iid vector error process such that $E(\boldsymbol{\eta}_t \boldsymbol{\eta}'_t) = \mathbf{I}$, $\boldsymbol{\omega} = (\omega_1, \ldots, \omega_N)'$ is a vector containing the ALOs' sizes and $I_T(t) = 1$ for $t \in T$ and 0 otherwise, representing the presence of ALOs at a given set of times T. ALOs can occur simultaneously at the same time t or not and their sizes can coincide or not.
Note that the conditional covariance matrix $\mathbf{H}_{t+1}$ depends on the past information through $\boldsymbol{\varepsilon}_t$ and $\mathbf{H}_t$, but not $\mathbf{y}_t$. Since the effect of the outlier is only in $\mathbf{y}_t$, the conditional covariance matrix will not be affected by this type of outliers. Indeed ALOs only affect the level of the series. This is true for all multivariate GARCH models.

2.2 Additive volatility outliers

Additive volatility outliers (AVOs) affect both the level of the time series of financial returns and their volatility (see Doornik and Ooms, 2005; Grané and Veiga, 2010). In this context, the conditional mean equation of the multivariate GARCH model becomes:

$$\begin{aligned} \mathbf{y}_t &= \boldsymbol{\varepsilon}_t \\ \boldsymbol{\varepsilon}_t &= \boldsymbol{\omega} \cdot I_T(t) + \mathbf{H}_t^{1/2} \boldsymbol{\eta}_t \end{aligned} \tag{5}$$

where $\boldsymbol{\eta}_t$, $\boldsymbol{\omega}$ and $I_T(t) = 1$ are defined as in Section 2.1.
In contrast to ALOs, the effect of AVOs in $\mathbf{y}_t$ is through the term $\boldsymbol{\varepsilon}_t$ which indeed affects the conditional covariance matrix $\mathbf{H}_{t+1}$. This means that the values of the returns following the outlier occurrence will also be affected, since the conditional covariance matrix has been modified by the outlier. In order to highlight this behavior we are going to focus in the D-BEKK model.

Let $\{\mathbf{y}_t\}$ be a vector stochastic process following the D-BEKK model described by equation (1) and $\{\mathbf{y}_t^*\}$ a vector stochastic process following the D-BEKK model contaminated with an AVO at time s. Let $\mathbf{H}_t^*$ and $\boldsymbol{\varepsilon}_t^*$ denote the conditional covariance matrix and the vector of errors for the contaminated process $\{\mathbf{y}_t^*\}$, respectively. Using this notation, equation (1) for the process $\{\mathbf{y}_t^*\}$ is:

$$\mathbf{H}_t^* = \mathbf{C}\mathbf{C}' + \mathbf{A}'\boldsymbol{\varepsilon}_{t-1}^*\boldsymbol{\varepsilon}_{t-1}^{*'}\mathbf{A} + \mathbf{B}'\mathbf{H}_{t-1}^*\mathbf{B}.$$

At time s, when the outlier occurs, $\boldsymbol{\varepsilon}_s^* = \boldsymbol{\omega} + \boldsymbol{\varepsilon}_s$. Hence, at time $s+1$, the conditional covariance matrix of the process $\{\mathbf{y}_t^*\}$ will get contaminated. That is:

$$\mathbf{H}_{s+1}^* = \mathbf{H}_{s+1} + \mathbf{A}'(\boldsymbol{\omega}\boldsymbol{\omega}' + \boldsymbol{\omega}\boldsymbol{\varepsilon}_s + \boldsymbol{\varepsilon}_s\boldsymbol{\omega}')\mathbf{A}.$$

Note that, after time $s+1$, the conditional covariance matrix of the process $\{\mathbf{y}_t^*\}$ will remain different than that of the non–contaminated process $\{\mathbf{y}_t\}$, since it is affected by both the second and the third terms of equation (1). It is easy to see that the third term is affected by the outlier since it ultimately depends on $\mathbf{H}_{t-1}^*$. The second term depends on $\boldsymbol{\varepsilon}_t^*$, whose covariance is actually $\mathbf{H}_t^*$, which is hence different from the non-contaminated vector of errors $\boldsymbol{\varepsilon}_t$.

Regarding, the CCC and DCC models the conditional covariance matrix $\mathbf{H}_t$ is also affected by the AVO, since it depends on the conditional variances obtained with the univariate GARCH models, that are as well affected by the AVO (see Grané and Veiga, 2010).

3 Effects of additive outliers on the correlations: A simulation study

In the univariate literature it is well know that outliers can affect the estimation of the parameters (Fox, 1972; Van Dijk et al., 1999; Verhoeven and McAleer, 2000), the tests of conditional homoscedasticity (Carnero et al., 2007; Grossi and Laurini, 2009), the out-of-sample volatility forecasts (Ledolter, 1989; Chen and Liu, 1993; Franses and Ghijsels, 1999; Grané and Veiga, 2009), the estimated volatilities (Carnero et al., 2012) and the risk measures (Grané and Veiga, 2014). However, there are few studies devoted to analyze the effects of outliers on the estimated correlations using multivariate GARCH models.

In this section we contribute in this line by implementing an intensive simulation study. It involves single, multiple and patches of additive level outliers and additive volatility outliers included in the conditional variance–covariance equations of the models described in Section 2.2. The frequency of the simulations is daily, $N = 2$ and the parameters used are: $\{\text{vec}(\mathbf{C}) = (0.053, 0.042, 0, 0.020)', \text{diag}(\mathbf{A}) = (0.161, 0.164)', \text{diag}(\mathbf{B}) = (0.983, 0.981)'\}$ for the D-BEKK model, $\{\boldsymbol{\alpha}_0 = (0.010, 0.013), \boldsymbol{\alpha}_1 = (0.049, 0.067), \boldsymbol{\beta}_1 = (0.940, 0.926), \rho_{12} = -0.606)\}$ for the CCC model and $\{\boldsymbol{\alpha}_0 = (0.010, 0.013), \boldsymbol{\alpha}_1 = (0.049, 0.067), \boldsymbol{\beta}_1 = (0.940, 0.926), \alpha = 0.015, \beta = 0.981\}$ for the DCC model, which are chosen by fitting the models to real time series of financial returns including commodities such as oil.[2] The multivariate GARCH models are estimated by Quasi-Maximum Likelihood (QML) with the G@RCH 6.0 package.

The outliers are placed randomly across the series and each scenario involves 1000 replications. In general, they are placed in the same position in the two series to mimic the behavior of financial markets. We proceed by describing all the considered situations:

[2] $\boldsymbol{\alpha}_0$, $\boldsymbol{\alpha}_1$, $\boldsymbol{\beta}_1$ are the vectors of parameters of the univariate GARCH(1,1) models (see Grané and Veiga, 2010, for details on these models). The simulation results are also robust to the choice of the parameter values.

- One isolated ALO of two different sizes $\omega_1 = 5\sigma_{y_1}, 10\sigma_{y_1}; \omega_2 = 5\sigma_{y_2}, 10\sigma_{y_2}$, respectively, in simulated series from a CCC, DCC and a D-BEKK(1,1,1) models with normal distributed errors. For each outlier size, the sample sizes considered are $n = 1000, 3000, 5000$.

- Multiple ALOs of sizes $\omega_1 = 5\sigma_{y_1}, 10\sigma_{y_1}; \omega_2 = 5\sigma_{y_2}, 10\sigma_{y_2}$, respectively, in simulated series from a CCC, DCC and a D-BEKK(1,1,1) models with normal distributed errors. For each outlier size, the sample sizes considered are $n = 1000, 3000, 5000$.

- Patches of three ALOs of sizes $\omega_1 = 5\sigma_{y_1}, 10\sigma_{y_1}; \omega_2 = 5\sigma_{y_2}, 10\sigma_{y_2}$, respectively, in simulated series from a CCC, DCC and a D-BEKK(1,1,1) models with normal distributed errors. The beginning of the patch is placed randomly. For each outlier size, the sample sizes considered are $n = 1000, 3000, 5000$.

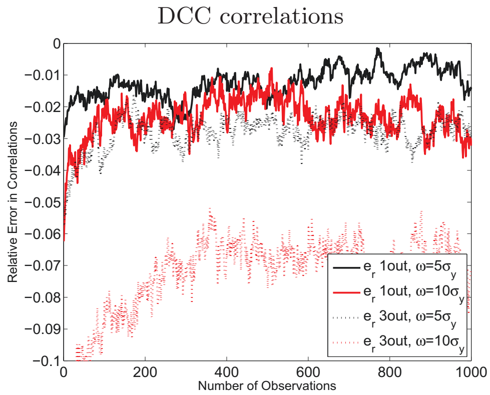
- One isolated AVO of two different sizes $\omega_1 = 25\sigma_{y_1}, 50\sigma_{y_1}; \omega_2 = 25\sigma_{y_2}, 50\sigma_{y_2}$, respectively, in simulated series from a CCC, DCC and a D-BEKK(1,1,1) models with normal distributed errors. For each outlier size, the sample sizes considered are $n = 1000, 3000, 5000$.

Table 1: Relative bias in the estimated correlations obtained from a CCC model with errors following normal distributions from 1000 simulated series of size n that include outliers of different magnitudes.

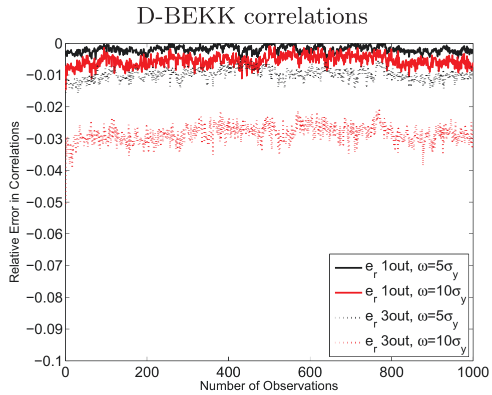
	n	Estimated Correlation	Relative Bias		n	Estimated Correlation	Relative Bias
1 ALO	1000	-0.5987	-0.013	3 ALOSs	1000	-0.5892	-0.028
$\boldsymbol{\omega} = 5\boldsymbol{\sigma}_{\mathbf{y}}$	3000	-0.6042	-0.004	$\boldsymbol{\omega} = 5\boldsymbol{\sigma}_{\mathbf{y}}$	3000	-0.6007	-0.010
	5000	-0.6051	-0.002		5000	-0.6017	-0.008
1 ALO	1000	-0.5872	-0.032	3 ALOs	1000	-0.5545	-0.086
$\boldsymbol{\omega} = 10\boldsymbol{\sigma}_{\mathbf{y}}$	3000	-0.5970	-0.016	$\boldsymbol{\omega} = 10\boldsymbol{\sigma}_{\mathbf{y}}$	3000	-0.5810	-0.042
	5000	-0.6012	-0.009		5000	-0.5902	-0.027
Patch of	1000	-0.5972	-0.015	1 AVO	1000	-0.5614	-0.074
3 ALOs	3000	-0.6031	-0.006	$\boldsymbol{\omega} = 25\boldsymbol{\sigma}_{\mathbf{y}}$	3000	-0.5805	-0.043
$\boldsymbol{\omega} = 5\boldsymbol{\sigma}_{\mathbf{y}}$	5000	-0.6041	-0.004		5000	-0.5847	-0.036
Patch of	1000	-0.5839	-0.037	1 AVO	1000	-0.5318	-0.123
3 ALOs	3000	-0.5959	-0.017	$\boldsymbol{\omega} = 50\boldsymbol{\sigma}_{\mathbf{y}}$	3000	-0.5627	-0.072
$\boldsymbol{\omega} = 10\boldsymbol{\sigma}_{\mathbf{y}}$	5000	-0.5999	-0.011		5000	-0.5642	-0.070
No outliers	1000	-0.6064					
	3000	-0.6065					
	5000	-0.6064					

Table 1 shows the estimated correlations (each reported value is the sample mean computed on 1000 values) for the CCC model and the relative biases (or relative errors) with respect to the estimated correlations in the absence of outliers. The three last lines correspond to the estimated correlations in the absence of outliers. Figures 1–3 contain the relative errors of DCC and D-BEKK models for different sample sizes. In particular, in Figure 1 we plot the

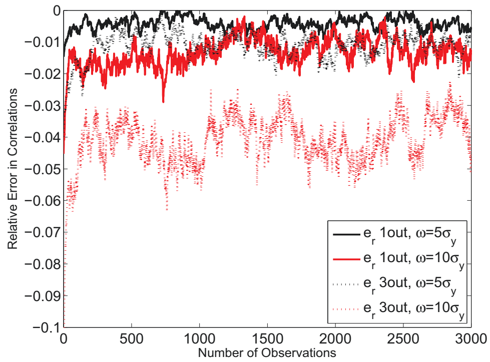
Figure 1: Relative bias in the estimated correlations obtained from a (a) DCC model and a (b) D-BEKK model with errors following normal distributions from 1000 simulated series of size n that include ALOs of different magnitudes.



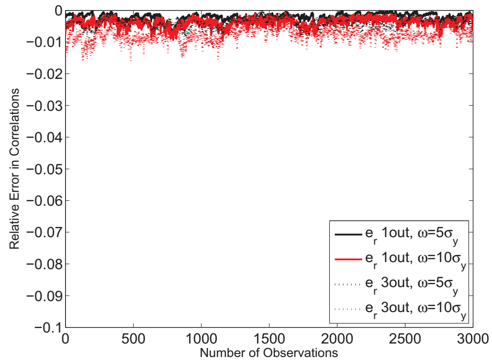
(a1) DCC, $n = 1000$



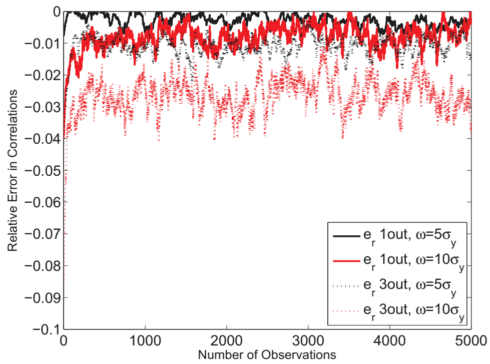
(b1) D-BEKK, $n = 1000$



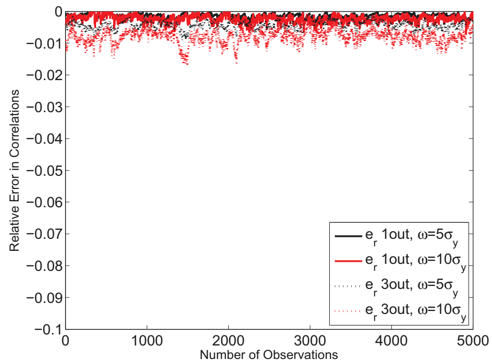
(a2) DCC, $n = 3000$



(b2) D-BEKK, $n = 3000$



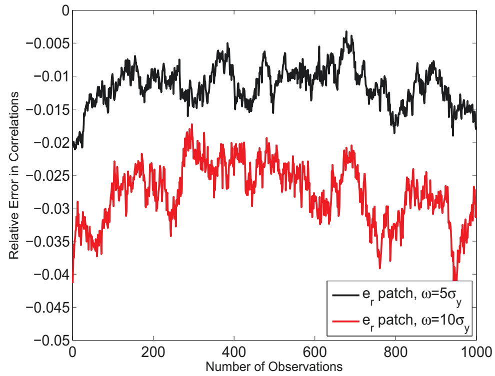
(a3) DCC, $n = 5000$



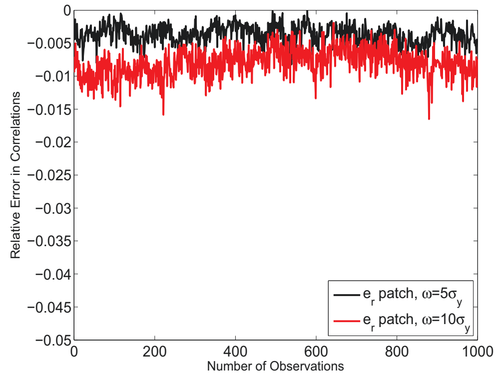
(b3) D-BEKK, $n = 5000$

relative bias obtained in the estimation of the correlations using DCC and D-BEKK models for the case of isolated ALOs, whereas Figures 2–3 correspond, respectively, to patches of ALOs and 1 isolated AVO. For each time t (going from 1 to N), the plotted value is the

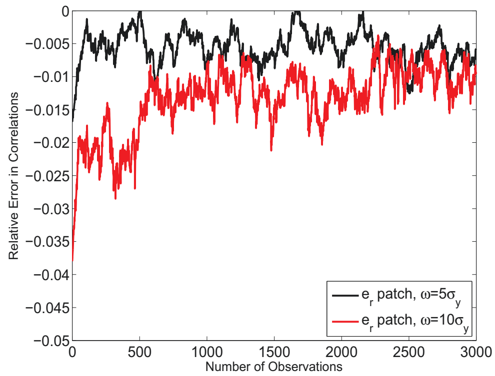
Figure 2: Relative bias in the estimated correlations obtained from a (a) DCC model and a (b) D-BEKK model with errors following normal distributions from 1000 simulated series of size n that include patches of different magnitudes.



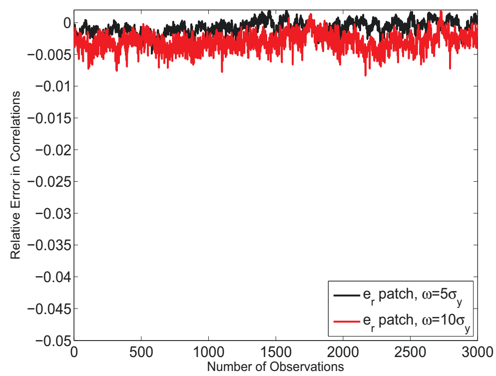
(a1) DCC, $n = 1000$



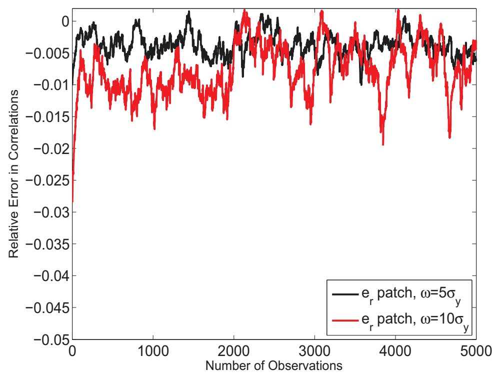
(b1) D-BEKK, $n = 1000$



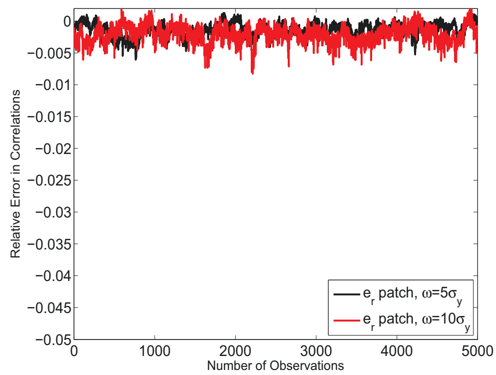
(a2) DCC, $n = 3000$



(b2) D-BEKK, $n = 3000$



(a3) DCC, $n = 5000$

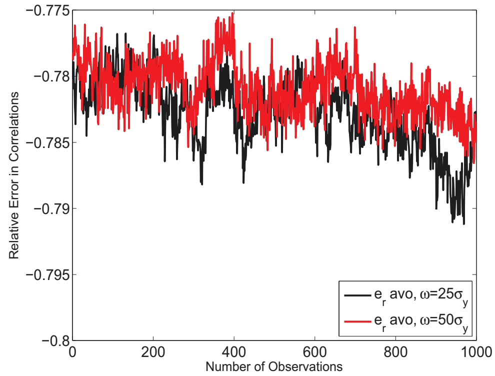


(b3) D-BEKK, $n = 5000$

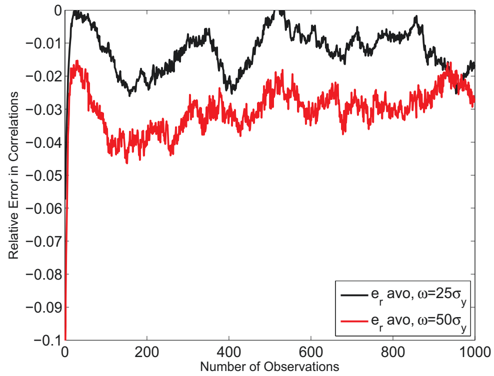
sample mean computed from 1000 replications.

From Table 1 and Figures 1–3 we can observe that the estimated correlations are affected by the presence of outliers and the relative errors are higher the higher is the magnitude of the outlier, the higher the number of outliers included in the simulated series and the smaller the

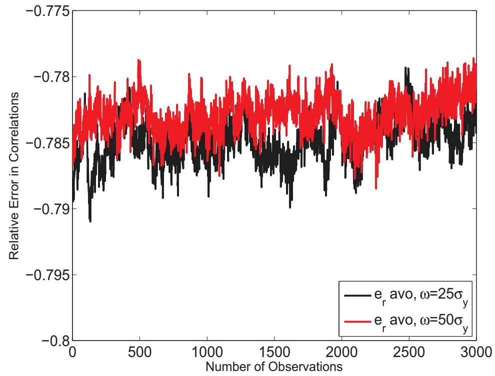
Figure 3: Relative bias in the estimated correlations obtained from a (a) DCC model and a (b) D-BEKK model with errors following normal distributions from 1000 simulated series of size n that include 1 AVO of different magnitudes.



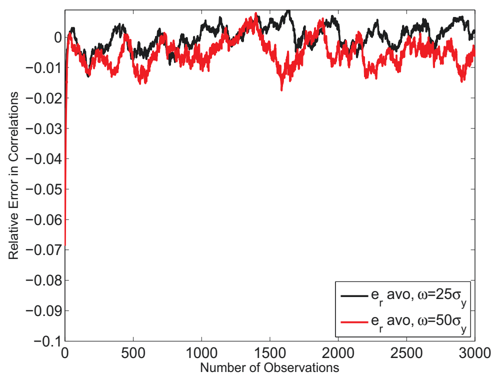
(a1) DCC, $n = 1000$



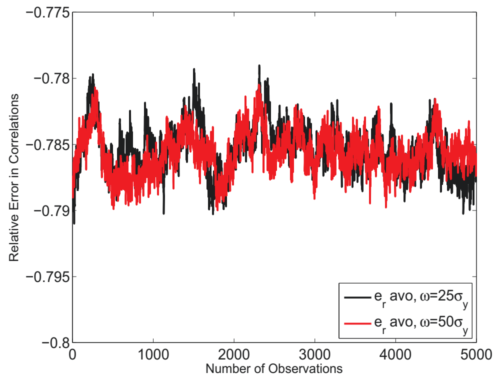
(b1) D-BEKK, $n = 1000$



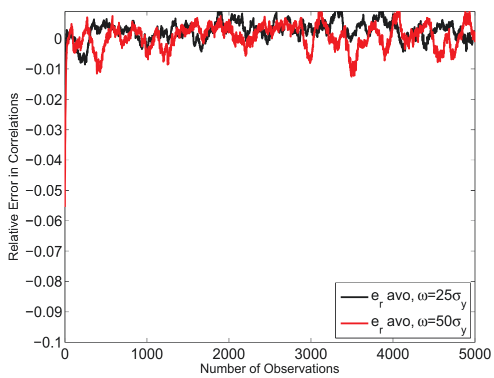
(a2) DCC, $n = 3000$



(b2) D-BEKK, $n = 3000$



(a3) DCC, $n = 5000$



(b3) D-BEKK, $n = 5000$

sample sizes of the simulated time series. Moreover, the biases in the correlations are higher for the DCC model in comparison to the CCC and D-BEKK models. In particular, the latter seems to be more robust to the presence of outliers since the correlations present small relative errors over the sample size. Finally, another conclusion is that additive outliers (level or volatility) bias the estimated correlations towards zero for the three considered models.

4 Wavelet-based detection procedure

In Grané and Veiga (2010) a general outlier detection method based on wavelets was introduced for the univariate case. The proposal was evaluated through an intensive simulation study, whose results show its effectiveness in detecting isolated level outliers, patches of level outliers and volatility outliers in large univariate financial time series. Moreover, the method was proven to be very reliable, since it detects a significantly smaller number of false outliers compared to other competitive methods.

In this work, it is of our interest to extend to the multivariate case a procedure with as good properties as the method proposed by Grané and Veiga (2010) for the univariate context, that is, effectiveness and reliability, and also of feasible implementation in large data sets. A possible way to proceed is to translate the multivariate problem to a univariate setting. This is achieved by applying the random projection method. In Cuesta-Albertos et al. (2006) and Cuesta-Albertos et al. (2007) some theoretical results were developed in the context of functional data (also of application whenever the data can be considered as independent and identically distributed draws of a stochastic process taking values in a Hilbert space). Cuesta-Albertos et al. (2006) intuitively describe the random projection method in the following way. Imagine we have to deal with a problem related to d-dimensional objects. The random projection method consists of choosing, at random, a subspace of dimension k (where k is low compared to d), solve the problem in the k-dimensional subspace and translate the solution to the original (d-dimensional) space. In practice, $k = 1, 2$, which is exactly contrary to the Projection Pursuit paradigm, avoiding implementation problems due to high-dimensionality. Our procedure is to be applied to the residuals of multivariate GARCH models or other multivariate volatility models such as the stochastic volatility models, although in this paper we focus our attention on the former class of models, to detect if some observations are outliers for these models. Our procedure is the first, to our knowledge, to detect outliers in multivariate volatility models and can also be interpretable as a miss-specification test to the model since if outliers are detected in the series of residuals this may suggest the rejection of the model.

Next in Section 4.1 we describe the proposed method to detect outliers and evaluate its performance in Section 4.2.

4.1 The procedure

The procedure we propose is based on the detail wavelet coefficients resulting from the discrete wavelet transform (DWT) of a univariate series of (standardized) residuals. The procedure starts by fitting a multivariate GARCH model and obtaining the series of multivariate residuals. Note that our proposal is model-dependent, but general enough to cope with a wide variety of models. The next step consists in transforming the multivariate series of residuals into univariate series to which DWT will be applied. Here we consider two different cases. Conditional correlation models, such as CCC and DCC, are based on the decomposition of the conditional covariance matrix. Hence, for these models, the decomposition property suggests that it is enough to consider only the univariate marginals. However, for models that do not have this property, as it is the case of the D-BEKK model, in addition to the marginals, we consider one randomly chosen projection (see Cuesta-Albertos et al., 2006). DWT is applied

to each of the univariate series under consideration and outliers are identified as those observations in the original series whose detail coefficients are greater (in absolute value) than a certain threshold.
In the context of financial return time series it is quite common to assume an underlying model for the data. Then, if the fitted model has captured the structure of the data, the residuals are supposed to be independent and identically distributed random variables following a specified distribution. Our proposal is to use the following test statistic: the maximum of the detail wavelet coefficients (in absolute value) resulting from the DTW of a univariate series of (standardized) residuals. If the univariate series under consideration is obtained as the marginal of the multivariate one, the distribution of the test statistic reported in Grané and Veiga (2010) for the univariate case is still valid. For the case in which the univariate series is obtained as a random projection the distribution of the test statistic is obtained via Monte Carlo, analogously. In practice, we find that in order to detect isolated ALOs it suffices to work with the first level detail wavelet coefficients. However, if there are patches of ALOs or isolated AVOs, it is necessary to use both first level and second level detail wavelet coefficients. From the simulation study (see Section 4.2) we believe that a reasonable threshold to use in the detection of isolated ALOs is the 95-th percentile, whereas for the detection of patches of ALOs and isolated AVOs the 90-th percentile is more useful. An analogous situation occurred in the univariate case.
Since in the multivariate case we are considering more than one series, the thresholds proposed in Grané and Veiga (2010) for the univariate case are not directly applicable and the union-intersection principle (Roy, 1953) with Bonferroni correction is applied. As a reference, Table 2 contains the values of the thresholds after applying the Bonferroni correction for bivariate series. The third, fourth, seventh and eighth columns correspond to the thresholds for the case in which only the marginals are considered and the fifth, sixth, ninth and tenth columns, to the case in which both the marginals and a random projection are considered. The thresholds are shown for two different significance levels $\alpha = 0.05$ and 0.10, three different sample sizes $n = 1000, 3000$, and 5000 and two different error distributions.

Table 2: Threshold values: Percentiles of the distribution of the test statistics (with Bonferroni correction).

		Gaussian distributed errors				Student's t distributed errors			
		only marginals		marginals and random projection		only marginals		marginals and random projection	
	n	1st level	2nd level	1st level	2nd level	1st level	2nd level	1st level	2nd level
$\alpha = 0.05$	1000	4.0595	3.8827	4.1386	3.9731	5.2583	4.6062	5.2390	4.6399
	3000	4.2995	4.1437	4.3885	4.2383	5.7469	5.0101	5.7214	5.0092
	5000	4.4062	4.2664	4.5027	4.3503	6.0131	5.1269	5.9162	5.1953
$\alpha = 0.10$	1000	3.7216	3.5280	3.9944	3.8207	4.9470	4.3384	4.9215	4.4039
	3000	3.8965	3.7114	4.2319	4.0873	5.4087	4.7086	5.3850	4.7845
	5000	4.2620	4.0992	4.3607	4.2012	5.6332	4.9015	5.6013	4.9383

4.2 Performance of the procedure: A simulation study

In this section we present the results of an intensive simulation study to assess the performance of our detection proposal. In this study, we simulate the contaminated and no–contaminated multivariate series as described in Section 3 and, additionally, we include D-BEKK, CCC and DCC models with Student-t distributed errors.[3]

We apply the detection method described in Section 4.1 where the assumed model is the true model used to generate the series. The results are shown in Tables 3–4. The measures used in the performance study are the percentage of times that the localization of the outliers is correctly detected (columns 3-5) and the percentage of false outliers (columns 6-8). The threshold values used in the study are contained in Table 2.

Table 3: Percentage of correct detection of outliers and percentage of false outliers in 1000 replications of size n for a multivariate GARCH model with errors following a normal distribution.

		% of correct detections			% of false outliers		
	n	D-BEKK	CCC	DCC	D-BEKK	CCC	DCC
1 ALO	1000	43.8	77.1	77.2	0.004	0.005	0.005
$\boldsymbol{\omega} = 5\boldsymbol{\sigma}_{\mathbf{y}}$	3000	38.7	76.2	75.3	0.001	0.001	0.001
	5000	36.1	69.8	70.8	0.001	0.001	0.001
1 ALO	1000	99.1	100.0	100.0	0.004	0.004	0.048
$\boldsymbol{\omega} = 10\boldsymbol{\sigma}_{\mathbf{y}}$	3000	99.3	99.9	99.9	0.001	0.001	0.001
	5000	99.3	99.9	99.8	0.001	0.001	0.001
3 ALOs	1000	36.7	69.6	68.9	0.003	0.004	0.004
$\boldsymbol{\omega} = 5\boldsymbol{\sigma}_{\mathbf{y}}$	3000	36.5	71.2	71.1	0.001	0.001	0.001
	5000	36.1	71.5	71.4	0.001	0.001	0.001
3 ALOs	1000	96.5	97.8	97.8	0.002	0.005	0.005
$\boldsymbol{\omega} = 10\boldsymbol{\sigma}_{\mathbf{y}}$	3000	97.8	98.9	98.8	0.001	0.001	0.001
	5000	97.8	99.1	98.9	0.001	0.001	0.001
Patch of 3 ALOs	1000	26.4	20.5	20.4	0.0001	0	0
$\boldsymbol{\omega} = 5\boldsymbol{\sigma}_{\mathbf{y}}$	3000	30.5	18.8	18.8	0	0	0
	5000	33.1	17.7	18.9	0.00002	0	0
Patch of 3 ALOs	1000	73.2	89.2	88.5	0	0	0
$\boldsymbol{\omega} = 10\boldsymbol{\sigma}_{\mathbf{y}}$	3000	70.4	88.1	87.6	0	0	0
	5000	70.5	86.0	85.3	0.00002	0	0
1 AVO	1000	24.2	66.4	95.6	0.001	0.0001	0
$\boldsymbol{\omega} = 25\boldsymbol{\sigma}_{\mathbf{y}}$	3000	24.1	66.8	94.8	0.0003	0	0
	5000	24.4	66.5	95.6	0.0002	0	0
1 AVO	1000	52.2	87.0	99.6	0.004	0	0
$\boldsymbol{\omega} = 50\boldsymbol{\sigma}_{\mathbf{y}}$	3000	55.6	88.0	99.3	0.002	0	0
	5000	53.9	88.8	99.8	0.001	0	0
No outliers	1000				0.004	0.006	0.005
	3000				0.001	0.001	0.001
	5000				0.001	0.001	0.001

The detection rate is greater for models with Gaussian errors. From Table 3 we can see that when the magnitude of the outlier is $\boldsymbol{\omega} = 10\boldsymbol{\sigma}_{\mathbf{y}}$, the procedure detects more than 96% of

[3]The parameters used are: $\{\text{vec}(\mathbf{C}) = (0.106, 0.110, 0, 0.0371)', \text{diag}(\mathbf{A}) = (0.0571, 0.050)', \text{diag}(\mathbf{B}) = (0.983, 0.985)'\}$ for the D-BEKK model, $\{\boldsymbol{\alpha}_0 = (0.010, 0.013), \boldsymbol{\alpha}_1 = (0.049, 0.067), \boldsymbol{\beta}_1 = (0.740, 0.759), \rho_{12} = 0.506\}$ for the CCC model and $\{\boldsymbol{\alpha}_0 = (0.106, 0.110, 0.0371), \boldsymbol{\alpha}_1 = (0.0571, 0.050), \boldsymbol{\beta}_1 = (0.740, 0.759), \alpha = 0.015, \beta = 0.781\}$ for the DCC model. Student-t distributed errors with 7 d.f.

the isolated outliers, reaching the 100% in two cases. When the magnitude of the outlier is relatively small, $\boldsymbol{\omega} = 5\boldsymbol{\sigma}_{\mathbf{y}}$, the detection rate goes from 36% to 43% for the D-BEKK model and from 68% and 77% for the CCC and DCC models. Regarding patches and volatility outliers, the detection rate also increases with the size of the outlier and it ranges from 24.1% (AVO and D-BEKK) to 99.8% (AVO and DCC). Finally, the percentage of false positives is at most 0.001% in 80% of the cases and under 0.007% in the rest (the only exception is the DCC model for $\boldsymbol{\omega} = 10\boldsymbol{\sigma}_{\mathbf{y}}, n = 1000$). Concerning models with Student's t distributed errors, from Table 4 we observe that, for example, when the magnitude of the outlier is $\boldsymbol{\omega} = 10\boldsymbol{\sigma}_{\mathbf{y}}$, the procedure detects from 70.9% to 99.2% of isolated ALOs. As expected, the detection rate is low when $\boldsymbol{\omega} = 5\boldsymbol{\sigma}_{\mathbf{y}}$, since it is difficult to distinguish small size outliers from the thick tail of Student's t distribution. The percentage of false positives is still very small, being at most 0.006% in more than 77% of the cases. These results lead us to conclude that the method is very reliable.
In general, the percentage of correctly detected outliers is smaller for the D-BEKK model than for CCC and DCC and this is confirmed by the results presented in Section 3, that the effect of outliers in the estimation of the correlations is lower for D-BEKK model than for the CCC or DCC models.

These results show the reliability of the outlier detection method for bivariate series. A natural question arises for higher-dimensional cases: is one random projection enough for detecting outliers or should the number of random projections be increased with N? As shown in what follows, our results suggest that there is no need to increase the number of random projections considered in the algorithm.

One or more random projections? We focus now on analyzing the performance of our procedure for $N = 10$ series and the D-BEKK model.[4] In particular, we are interested in studying whether increasing the number of random projections may increase the percentage of correctly detected outliers. In this case, threshold values are computed as suggested by Benjamini and Yekutieli (2001), instead of Bonferroni correction, which is too conservative. Results are contained in Table 5. As before, the measures used are the percentage of times that the localization of the outliers is correctly detected and the percentage of false outliers. The number of random projections is shown in the first column. We analyze here the case of 1 ALO of sizes $\omega = 5\boldsymbol{\sigma}_{\mathbf{y}}, 10\boldsymbol{\sigma}_{\mathbf{y}}$. For $\omega = 10\boldsymbol{\sigma}_{\mathbf{y}}$ the proportion of correct detections stays constant when the number of random projection is increased, whereas for $\omega = 5\boldsymbol{\sigma}_{\mathbf{y}}$ the increase is very low (0.2 percentage points). In contrast, the percentage of false outliers worsen with an increase on the number of random projections, hence suggesting that it is not advisable to use more than one random projection for large values of N.

5 Empirical application

In this section we analyze three financial time series of returns to illustrate the performance of our method on real data. The series considered are three of the most important indices of the U.S. stock market, such as Nasdaq, NYSE and S&P500. The data was collected from Yahoo

[4]Regarding the computational burden of this simulation study, we want to remark that estimating 1000 times the D-BEKK model for 10 series took approximately one week in an ordinary computer.

Table 4: Percentage of correct detection of outliers and percentage of false outliers in 1000 replications of size n for a multivariate GARCH model with errors following a Student's t distribution.

		% of correct detections			% of false outliers		
	n	D-BEKK	CCC	DCC	D-BEKK	CCC	DCC
1 ALO	1000	12.5	7.7	7.8	0.0130	0.0082	0.0083
$\boldsymbol{\omega} = 5\boldsymbol{\sigma}_{\mathbf{y}}$	3000	11.5	3.2	3.2	0.0082	0.0055	0.0056
	5000	11.2	2.1	2.1	0.0067	0.0050	0.0049
1 ALO	1000	92.2	98.8	99.0	0.0127	0.1080	0.0067
$\boldsymbol{\omega} = 10\boldsymbol{\sigma}_{\mathbf{y}}$	3000	81.7	98.4	98.4	0.0081	0.0050	0.0050
	5000	73.3	99.2	99.1	0.0068	0.0047	0.0047
3 ALOs	1000	12.3	5.6	5.6	0.0113	0.0568	0.0566
$\boldsymbol{\omega} = 5\boldsymbol{\sigma}_{\mathbf{y}}$	3000	10.3	2.5	2.5	0.0077	0.0050	0.0050
	5000	11.4	2.2	2.2	0.0066	0.0047	0.0048
3 ALOs	1000	88.2	93.0	93.3	0.0103	0.0062	0.0059
$\boldsymbol{\omega} = 10\boldsymbol{\sigma}_{\mathbf{y}}$	3000	78.8	97.9	97.8	0.0075	0.0542	0.0042
	5000	70.9	98.1	98.2	0.0065	0.0043	0.0043
Patch of 3 ALOs	1000	24.0	1.7	1.9	0	0	0.0001
$\boldsymbol{\omega} = 5\boldsymbol{\sigma}_{\mathbf{y}}$	3000	26.9	0.7	0.7	0.0001	0	0
	5000	26.6	0.1	0.1	0.0001	0.00004	0.00002
Patch of 3 ALOs	1000	52.3	77.8	77.8	0	0	0
$\boldsymbol{\omega} = 10\boldsymbol{\sigma}_{\mathbf{y}}$	3000	44.1	71.1	71.1	0.0001	0	0
	5000	41.1	63.2	63.3	0.0001	0.00002	0.00002
1 AVO	1000	3.2	46.3	70.1	0.0001	0	0
$\boldsymbol{\omega} = 25\boldsymbol{\sigma}_{\mathbf{y}}$	3000	3.3	47.1	66.6	0.0001	0.0001	0
	5000	2.8	44.4	66.3	0.0002	0	0
1 AVO	1000	15.0	75.1	94.5	0.0002	0	0
$\boldsymbol{\omega} = 50\boldsymbol{\sigma}_{\mathbf{y}}$	3000	17.7	75.5	94.0	0.0002	0	0
	5000	17.3	75.0	94.2	0.0002	0	0
No outliers	1000				0.0142	0.0092	0.0094
	3000				0.0082	0.0057	0.0058
	5000				0.0068	0.0050	0.0050

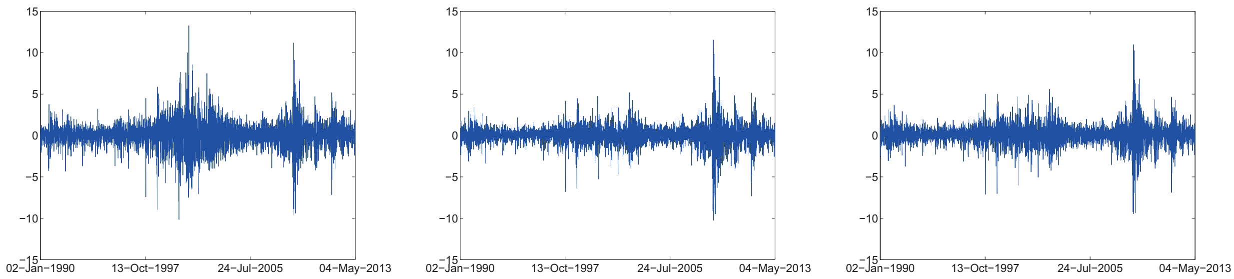
Table 5: Percentage of correct detection of outliers and percentage of false outliers in 1000 replications of size $n = 1000$ for a D-BEKK model with Gaussian distributed errors. Series contaminated with one ALO of two different sizes.

	1 ALO $\boldsymbol{\omega} = 5\boldsymbol{\sigma}_{\mathbf{y}}$		1 ALO $\boldsymbol{\omega} = 10\boldsymbol{\sigma}_{\mathbf{y}}$	
num. of random projections	% of correct detections	% of false outliers	% of correct detections	% of false outliers
1	21.4	0.0065	97.7	0.0241
2	21.4	0.0064	97.7	0.0178
5	21.4	0.0079	97.7	0.0212
10	21.6	0.0085	97.7	0.0280
20	21.6	0.0115	97.7	0.0377
50	21.6	0.0182	97.7	0.0672

Finance website (http://finance.yahoo.com) and spans the period of January 2, 1990–May 3, 2013.

Figure 4 depicts the three return series, $y_t = (\log p_t - \log p_{t-1}) \cdot 100$, where p_t is the value at time t of the corresponding index and Table 6 reports some summary statistics.

Figure 4: Returns in percentage for several financial time series.



(a) Nasdaq index (b) NYSE index (c) S&P500 index

Table 6: Descriptive statistics for daily stock index returns.

Stock index returns	Nasdaq	NYSE	S&P500
Mean	0.034	0.025	0.026
Variance	2.346	1.288	1.362
Skewness	-0.076*	-0.383*	-0.232*
Kurtosis	8.912*	13.718*	11.515*
KS_S	-2.376	-11.976	-7.266
KS_K	92.520	167.724	133.247

From Table 6, we observe that the three return series are negatively skewed and have significant kurtosis, ranging from 8.912 for NYSE to 13.718 for Nasdaq, which suggests the existence of some outliers. It is known that this type of observations in time series leads to fat tail distributions, and some outlier detection methods, specially in the multivariate context, are based on this information (see for example Peña and Prieto, 2001; Galeano et al., 2006). Table 6 also contains the results of the Kiefer and Salmon (1983) test, which is a formal test of normality in the context of conditional heteroscedastic series.[5] The test confirms the non Gaussianity of the three return series.

Next, we estimate the three multivariate GARCH models considered in this work: the CCC, the DCC and the D-BEKK models with Gaussian and Student-t distributed errors, and we proceed by applying our method to detect outliers.[6] This procedure can also be interpreted as a miss-specification test to the models. If one detects outliers in the residuals this may suggest that the estimated models are not appropriate to model the series. In fact, our procedure

[5]The Kiefer and Salmon (1983) test is given by $KS_N = (KS_S)^2 + (KS_K)^2$, where $KS_S = \sqrt{\frac{T}{6}}\left[\frac{1}{T}\sum_{t=1}^{T} y_t^{*3} - \frac{3}{T}\sum_{t=1}^{T} y_t^*\right]$, $KS_K = \sqrt{\frac{T}{24}}\left[\frac{1}{T}\sum_{t=1}^{T} y_t^{*4} - \frac{6}{T}\sum_{t=1}^{T} y_t^{*2} + 3\right]$ and y_t^* are the standardized returns. If the distribution of y_t^* is conditional $N(0,1)$, then KS_S and KS_K are asymptotically $N(0,1)$ and KS_N is asymptotically $\chi^2(2)$.

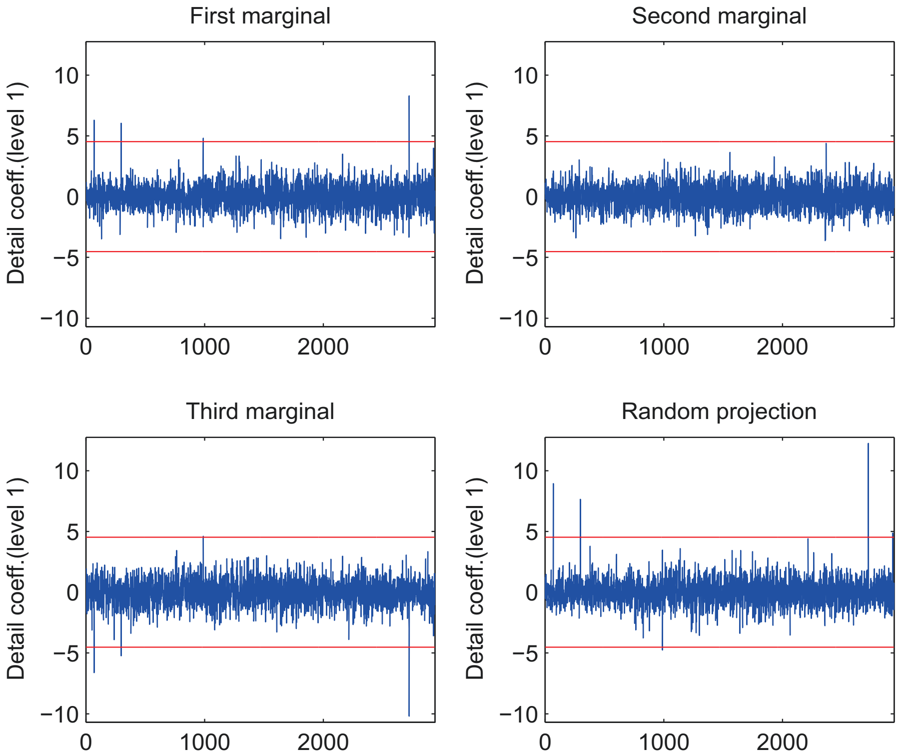
[6]The degrees of freedom of the Student-t distributions are considered endogenous and therefore estimated.

detects some outliers but they are different for the D-BEKK and the conditional correlation models. Results are shown in Table 7.

Table 7: Observations identified as possible outliers in the three series of stock market indices.

	Gaussian errors			Student's t-distributed errors		
	D-BEKK	CCC	DCC	D-BEKK	CCC	DCC
ALOs	137	475	475	137	-	-
	595	4324	4324	595	-	-
	1978	-	-	5447	-	-
	1979	-	-	-	-	-
	5447	-	-	-	-	-
Patches	5447	1978	1978	-	1978	1978

Figure 5: Graphical output of the wavelet-based procedure for the returns of Nasdaq, NYSE and S&P500 estimated with a D-BEKK model with Gaussian errors.



Regarding Gaussian errors, observation 137 corresponds to July 18, 1990. In July 1990 Iraq invaded Kuwait causing falls in stock markets. The second outlier is observation 475 that corresponds to November, 1991, when Nasdaq average prices fell over 4%, representing the first major correction in the post-crash October 19, 1987 era. Observation 595 (May 5, 1995) corresponds to a period of downturn of the financial markets that lasted 3 months. October 27, 1997 (observation 1978) is also detected as an outlier. In this day there was a mini crash caused by an economic crisis in Asia and a recovery in the following day (observation 1979). Another outlier is observation 4324 that corresponds to February 27, 2007, the day of the big decline in Chinese stocks and the news of the weakness in some key readings on the US economy. Finally, observation 5447 corresponds to August 10, 2011. In this month there was a sharp drop in stock prices across the United States, Middle East, Europe and Asia. This was caused by the fear of contagion of the European sovereign debt crisis to Spain and Italy. Regarding the detection of patches, the method detects one around the observation 5447 (August 10, 2011) for the D-BEKK and other around observation 1978 (October 27, 1997) for the CCC and the DCC models. Figure 5 shows a graphical output of the Matlab program, which corresponds to the analysis of the multivariate residuals obtained after fitting a D-BEKK model with Gaussian errors to Nasdaq, NYSE and S&P500 returns.

Looking at the detection results when considering Student-t distributed errors we observe that much less observations are detected as outliers, although if detected, they coincide with those detected when Gaussian errors where considered. Therefore, multivariate GARCH models with Student-t distributed errors are much more robust to extreme observations than Gaussian multivariate GARCH models.

According to our findings, our procedure seems to be quite effective in capturing the most important crashes in the three most important international stock markets and consequently it works as a reliable misspecification test based on residual-diagnostics.

6 Conclusion

A first contribution of this paper is the study of the impact of additive outliers (isolated level outliers, patches of level outliers and volatility outliers) on the estimation of correlations when fitting well known multivariate GARCH models via an intensive simulation study. The results of the Monte Carlo experiments show that correlations are considerably affected by the presence of outliers, the higher is the magnitude of the outlier, the higher the number of outliers included in the simulated series and the smaller the sample sizes of the simulated time series. Another finding is that, when the true correlation is negative, additive outliers of the same sign of that of the observations bias the estimated correlations towards zero. This represents the case of portfolios that include asset returns and commodity returns such as oil. Finally, outliers impact the conditional correlation models stronger than the diagonal-BEKK model.

A second contribution of this paper is the proposal of a general detection algorithm based on wavelets that can be applied to a large class of multivariate volatility models. This procedure can also be interpreted as a model miss-specification test since it is based on residual diagnostics. The effectiveness of our method is evaluated both with simulated and real data. The simulations report evidence that our proposal is both effective and reliable since it detects very few false outliers.

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